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Design and maintenance programme optimization for large infrastructure systems

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The design, construction and operation of civil infrastructure systems in modern society has become a critical issue worldwide. These processes cannot be restricted to solving just the mechanical problem. On the contrary, they should be designed within a framework that considers their life cycle, taking into account all costs in which the owner (or operator) will incur during the lifetime of the structure, and the benefits derived from its existence. This paper presents a strategy for optimizing the maintenance of technical facilities based on their life cycle within which deterioration and sudden failure due to extreme events may occur. A renewal model for the sequence of structural failures is used to define the objective function. The proposed model is useful for defining both an optimal maintenance policy (number of interventions and the time between them) and the design parameters. It can also be used for investment, planning and operation of new and existing facilities. The results have shown that a rational programme for maintenance and structural updating is essential for defining the efficiency of the investment in infrastructure projects.

Keywords: Optimization; Infrastructure systems; Maintenance; Reliability; Life cycle cost

1. Introduction

Civil infrastructure systems are critical assets for any country’s socioeconomic development. Designing these systems for a particular service life and maintaining them in operation has been recognized as a critical issue worldwide. The design of modern structural facilities cannot be restricted to solving just the mechanical problem. It should also consider the interaction between all involved parties (e.g. owners, designers, constructors, operators, regulatory agencies, etc.) and all the processes throughout the project’s life (e.g. planning design, construction, operation, etc.) as shown in figure 1. Integrating all these aspects is only possible by establishing a common framework for decision making, which is frequently defined in economic terms. Infrastructure systems should be designed taking into account all costs which the owner (or operator) will incur during the lifetime of the facility, and the benefits derived from its existence. Based on these considerations, it can be concluded that the traditional engineering view is moving towards a new paradigm which requires acting on the following directions: (1) from partial focus to holistic thinking; (2) from structure to process orientation; (3) from cost allocation to cost tracing; and (4) from deterministic to uncertainty management (Emblemsvag 2003). In summary, the development of modern infrastructure has to be built on integrating technical, as well as economic, concepts within a global framework of intergenerational responsibility, environmental protection and public welfare. It should provide safe and economically feasible solutions in the long term.

The life cycle of any project refers to the time lapse during which ‘someone’ invests resources and expects to obtain some benefit. It is expected that costs should be

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lower than benefits in the long run; otherwise it is not worth building the project. The life cycle cost can then be defined as ‘the total cost that is incurred, or may be incurred, in all stages of the project life cycle’ (Emblemsvag 2003). It is important to stress that life cycle cost analysis is a decision support tool for decision making and not an external financial reporting system.

As in industry, engineering products such as infrastructure facilities have to extend the cost evaluation from the simple ‘counting’ approach to the life cycle where value is created. Thus, the analysis should look forward in time, beyond the organization production costs. It should focus on the underlying drivers of business and economic performance. For large infrastructure projects, cost and benefit estimation is not easy to quantify since the social impact plays a significant part in the decision. Furthermore, the almost ‘infinite’ time design has shown that investment associated with interventions during the life span is several times higher than the original cost. Expected interventions are associated with, for instance, unexpected unfavourable events or deterioration processes. Then, life cycle cost analysis should handle all kinds of risks to the infrastructure that can incur losses. These risks range from classical engineering (failure of the structure) to business risks, which have recently shown to be a new focal point of corporate governance. Thus, for life cycle analysis to be effective, a proactive cost management approach has to be adopted and it should take risk and uncertainty into consideration to be really useful for decision making. Within this context, it is evident that cost management should ideally be expanded to risk-based cost management, as well as focus on total cost.

Life cycle engineering of large infrastructure systems is closely related to sustainability since it takes into account the long term (socioeconomic) consequences of a decision that has to be made today. Therefore, aspects related to environmental impact, socioeconomic development, inter-generational responsibility, in addition to all technical issues (i.e. safety and serviceability requirements) become relevant. Decisions have to be made based on a model that is able to combine these aspects and the effects of every one of them on the decision. Since the final decision has to be made considering socioeconomic factors, it is also required to consider for instance demographic patterns, economic indicators (Gross Domestic product, GDP and Life Quality index LQI) and discounting rates, which have to deal with aspects that are not easily quantifiable. A comprehensive analysis should be directed to solve a problem where the life cycle of any project should be balanced to obtain an optimum solution, which also reflects the welfare and long term sustainability of society. This view of the problem is particularly relevant when decisions are made about large infrastructure systems such as transport networks, water and power supply systems, oil refineries, etc., which are systems that do not have a finite lifetime span, and their implementation will have an impact on the welfare of future generations.

In life cycle analysis, the technical problem focuses around defining the interaction between three main processes: (1) structural deterioration (e.g. ageing or obsolescence); (2) the occurrence of unexpected extreme events (e.g. storms or floods); (3) the introduction of new technologies or changes in the environment that affect the performance and cost of the infrastructure.
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earthquakes or floods); and (3) the maintenance/rehabilitation program. The first aspect corresponds to the structure’s loss of functionality (also expressed in terms of safety) with time. It is usually a slow time-dependent process controlled by a safety/operation threshold specification. Examples of this case are structures in aggressive environments and subject to, for instance, sulphur or chloride penetration, and/or biodeterioration. Other examples are structures subject to phenomena such as fatigue or creep, e.g., offshore platforms and sea ports. The occurrence of unexpected or rare events which may dramatically affect the structure’s safety are commonly associated with both natural or manmade disasters. Their impact can be observed, for example, when evaluating the impact of hurricanes Katrina and Rita on the infrastructure systems of Louisiana and Texas (e.g., damage to electricity and transport networks). The third aspect, defining an appropriate maintenance program, is definitive when considering a long term investment policy. Existing evidence has shown that it is an important part of the integrity management process as a means of monitoring the performance of the infrastructure to ensure their safety and serviceability (Bucher and Frangopol 2005). It is widely accepted that maintenance can be classified into (1) preventive maintenance, which if it is not done, will cost more at a later stage to keep the structure in a safe condition; and (2) essential maintenance which is required to keep the structure safe (Yang et al. 2005). Maintenance measures require inspection planning, which is commonly based on general guidelines and engineering judgement; in most cases it is prescriptive and does not take into account the structure specific characteristics or make optimum use of the performance data observed. As a result of this strategy, a significant number of inspections are ineffective as they do not focus on the most critical areas, or do not use the most appropriate techniques. This results in uneven safety levels and wastage of limited maintenance resources (Onoufriou and Frangopol 2002). In recent years, there have been significant developments in the area of reliability-based inspection design and planning for complex structures, such as offshore and bridges, motivated by the need to optimize maintenance expenditure and achieve better safety levels at a lower cost.

Nowadays, most existing models combine economic considerations (associated with the benefits of the investment and the cost of construction, operation and the potential losses) by using a renewal process whereby the renewals are the maintenance actions that bring a component back into its original condition. Then, optimal investment decisions require a balance among the economic investment, the benefits derived from the existence of the structure and the expected consequences in case of failure. In other words, designing, erecting and maintaining structural facilities may be viewed as a decision problem where the maximum economic benefit of the life cycle of the project is achieved while the reliability requirements are simultaneously fulfilled at the decision point. This paper presents a strategy for optimizing the maintenance programme of any technical facility and the design parameters simultaneously. First of all, the paper will present the basic concepts of Life Cycle Based Design. It will then discuss a renewal model initially proposed by Rosenblueth and Mendoza (1971) and further developed by Rackwitz (2000). The proposed maintenance optimization model is then presented and discussed. Finally, results are illustrated with an example.

2. Reliability-based optimization

Any structure should be optimal with respect to the investment during its lifetime. For optimizing the investment, it is necessary to consider economic as well as mechanical variables. Consider the following objective function (Rackwitz 2000):

\[ Z(p) = B(p) - C(p) - D(p), \]

(1)

where \( B(p) \) is the benefit of the existence of the structure, \( C(p) \) is the construction cost and \( D(p) \) is the expected value of losses if there is failure. The vector parameter \( p \) accounts for all design parameters necessary to make the decision, e.g., the thickness of the pavement’s layers, the structural dimensions and the moduli of materials. The expected value of losses could be replaced by the cost of losses \( H(p) \) multiplied by the failure probability \( P_f(p) \). Equation (1) can then be modified to:

\[ Z(p) = B(p) - C(p) - H(p)P_f(p). \]

(2)

The decision about the project has to be made by analysing the function \( Z(p) \) at time \( t = 0 \). For the project to be feasible, the function \( Z(p) \) has to be positive. Therefore, the optimum design, construction and operation requirements are associated with the vector parameter \( p \) for which \( Z(p) \) is positive and maximum. In order to make the decision at time \( t = 0 \) all costs have to be discounted, as suggested by Rackwitz (2000), by using the following continuous capitalization function:

\[ \delta(t) = \exp(-\gamma t), \]

(3)

where \( \gamma \) is the interest rate and \( t \) the time in suitable units. Note that for small values of \( \gamma \) this expression is similar to \( \delta(t) = (1 + \gamma t)^{-1} \), which is widely used, and where \( \gamma' \) is the yearly discount rate. Note that equation (3) is a good approximation for \( \gamma \ll 1 \), (i.e., \( \gamma \approx \gamma' \)) (Rackwitz 2000). The owner or operator will probably take interest from the financial market; however, when investments are made in
the name of the public, the selection of such rates is very
difficult (Rackwitz et al. 2005). Furthermore, before
defining the discount rate, several issues such as how to
discount the cost of saving lives, the intergenerational
responsibility of society and the social inequality have to
be addressed. A comprehensive and interesting discus-
sion about interest rates can be found in Rackwitz et al.
(2005).

From equation (1), several important cases can be
considered: (1) failure upon construction; (2) structures
given up after first failure; and (3) systematic reconstruc-
tion. For planning maintenance and rehabilitation, the
third case is particularly interesting and will be discussed
in this paper. Cases (1) and (2) will not be considered, but
further details can be found in Rackwitz (2000) and
Rackwitz et al. (2005).

3. Time analysis considerations

In order to introduce a maintenance policy in the life cycle
analysis, only the case in which the structure is sys-
tematically reconstructed after a failure was studied. This
process of failure and repair is modelled as a renewal
process and $B(p)$, $C(p)$, and $D(p)$ from equation (1)
determined in terms of the economic parameters
(i.e. interest rates) and their corresponding behaviour
with time.

The construction cost, $C(p)$, is not discounted since it
is considered only at construction (i.e. at time $t=0$); it does
not change with time as it can be considered as an instant
cost. On the other hand, the expression for the benefit does
vary with time and can be expressed as:

$$B(t) = \int_0^T b \delta(t)dt = \frac{b}{\gamma} (1 - \exp(-\gamma T)), \quad (4)$$

where $b$ is the benefit expressed as the annual benefit, $\delta(t)$ is
the capitalization function (equation (3)), $T$ is the design
period, and $\gamma$ is the interest rate. It is interesting to stress
that when $T \to \infty$, the benefit has a simple solution based
on a Laplace transform, i.e. $B(t) = \frac{b}{\gamma}$.

Before considering the expected cost of the losses if there
is a failure, $D(p)$, it is important to make some considera-
tions related to the probabilistic nature of the failure and
repair process. If it is assumed that failure is associated
with the arrival of an extreme overload, the reliability problem
should consider a distribution function that describes the
arrival times of these events. Then, consider a structure
with design period $T$, where $m$ possible disturbances may
take place. The density function of the time to the $n$-arrival,
$f_n(t)$, is described by:

$$f_n(t) = \int_0^t f_{n-1}(t-\tau)f(\tau)d\tau, \quad (5)$$

If a Poisson process is used to describe the arrivals, a
simple formulation of $f(t)$ and $f_n(t)$ can be found:

$$f(t) = \lambda \exp(-\lambda t), \quad (6)$$

$$f_n(t) = \lambda^n \exp(-\lambda t) \frac{t^{n-1}}{(n-1)!}, \quad (7)$$

where $\lambda$ is the occurrence rate of the disturbance (i.e.
intensity of the Poisson process). In some cases, it might be
necessary to use a modified renewal process to describe the
time of arrivals, in which the distribution function to first
event differs from all other arrivals. In this case the density
function of the first renewal (i.e. first failure), $g_1(p,t)$, can be
written as (Rackwitz 2000):

$$g_1(p,t) = \sum_{n=1}^{E_0(T)} \left[ P_f(p)(1 - P_f(p))^{n-1} \right] f_n(t), \quad (8)$$

where $P_f(p)$ is the facility’s probability of failure when an
event (i.e. overload) occurs; and $E_0(T)$ is the expected
number of events in a time window $T$. The solution for the
subsequent times between failures can be expressed as:

$$g(p,t) = \sum_{n=1}^{E_0(T)} \left[ P_f(p)(1 - P_f(p))^{n-1} \right] \phi_n(t), \quad (9)$$

where $\phi_n(t)$ is the density function of the time to the
$n$-arrival with $n > 1$. The upper limit of the sum represents
the number of disturbances evaluated, in this case it
is the expected value of $m$ in $T$. Note that if a simple
renewal process is considered instead of a modified one,$\phi_n(t) = f_n(t)$.

Then, based on equations (8) and (9) the failure cost can be
evaluated as (Rackwitz 2000):

$$D(p) = (C(p) + H(p)) \frac{\int_0^T g_1(p,t)\delta(t)dt}{1 - \int_0^T g(p,t)\delta(t)dt}, \quad (10)$$

The failure cost is divided into: (1) the reconstruction
cost (which is close to the actual construction cost); and
(2) the cost related to collateral losses due to failure $H$, e.g. demolition, material and human losses and
insurance payments. Rackwitz (2000) suggested that for
an infinite time design period ($T \to \infty$) the Laplace
transform can be used and a time-independent solution for $D(p)$
becomes:

$$D(p) = (H(p) + C(p)) \frac{f_1(\gamma)}{1 - f_1(\gamma)}, \quad (11)$$

where $f_1(\gamma)$ is the Laplace transform of the density function
of the time to the first arrival, and $f^*(\gamma)$ is the Laplace
transform of $f_n(t)$ for $n \neq 1$. Equations (4) and (10) are
time-dependent and, although their solution might be computationally expensive, they allow new temporary actions like maintenance to be introduced.

It is important to stress that the solution when using the Laplace transform for the benefit (i.e. \( B(t) = b/\gamma \)) and for \( D(p) \) (equation (11)) cannot be applied if non-constant discount rates are considered, as the convolution theorem no longer holds. A model to deal with this case has recently been proposed by Rackwitz et al. (2005). Therefore, in the discussion presented in the following sections the discounting rate will be taken as the long term average rate as suggested by Rackwitz for infrastructure systems.

### 3.1 Comparing time-dependent and independent cases

Based on the discussion presented in the previous section, and replacing equations (4) and (10) in equation (1), the objective function for systematic reconstruction can be rewritten as a time-dependent function:

\[
Z(p, T) = \frac{b}{\gamma} (1 - \exp(-\gamma T)) - C(p) - (C(p) + H(p)) \int_0^T g(t, p, \delta(t)) dt / \left( 1 - \int_0^T g(t, p, \delta(t)) dt \right).
\]

As suggested by Rackwitz (2000, 2001) a stationary Poisson process (i.e. \( f(t) = \lambda \exp(-\lambda t) \)) could be used to describe the arrivals of extreme overloads. Therefore, assuming that \( T \to \infty \), it is possible to define a time-independent objective function as:

\[
Z_R(p) = \frac{b}{\gamma} - C(p) - (C(p) + H(p)) \int_0^T g(t, p, \delta(t)) dt / \left( 1 - \int_0^T g(t, p, \delta(t)) dt \right),
\]

where \( \lambda \) is the rate of the Poisson process and \( \gamma \) is the discounting rate. Note that time-dependency is removed by using Laplace transforms. This procedure is possible because of the form of the capitalization function used and the infinite design time period.

A comparison between equations (12) and (13) was made and the results are shown in figure 2. It can be observed that the lifetime required for equation (12) to converge into equation (13) basically depends on the discount rate. It was also found that it marginally depends on the Poisson occurrence rate of the events and on the order of magnitude of the failure probability.

As mentioned above, the behaviour of the objective function and the analysis of convergence is mainly controlled by how the discount rates are defined. Recent discussions placed within the context of sustainable development show that discounting for sustainability should at least be consistent with discounting for risk reduction investments. If they are chosen to reflect how the society becomes more wealthy, they must be calculated as the long term average of the economic growth per capita. In this case, values vary between 0.9% for Africa and 2.5% for USA and Canada (Rackwitz et al. 2005). Special attention should be given to volatile economies or economies subject to natural disasters where the reconstruction effects may cause a substantial change in the GDP from one year to another. Alternative ways for computing discounting rates can be found in the literature, e.g. the classical Ramseyan approach (Ramsey 1928) is widely used for optimal stable economic growth in perfect markets. In this model, the real market interest rate can be computed in terms of the preference of consumption, its elasticity and the consumption growth per capita. The results obtained by using this model imply discount rates of around 5% (Rackwitz et al. 2005). If these discounting values are considered the convergence is obtained after long time periods as shown in figure 2.

In this paper, only averaged financial discounting rates will be used. Given this assumption, it is widely accepted that there is a direct relation between the financial interest rates and a country’s level of development. Industrialized countries manage interest rates between 2% and 8%, while in moderate and low developed countries, interest rates may vary between 8−18% and 15−30% respectively. It can be observed in figure 2 that for an interest rate of \( \gamma = 20\% \) the solution of the equations converge after 15 years, while for \( \gamma = 10\% \) convergence is after 50 years and for \( \gamma = 5\% \) after 100 years. This implies that for highly industrialized countries (i.e. low interest rates) it is necessary to consider equation (12) instead of equation (13), while for moderate to low developed countries the difference between the results obtained from equations (12) and (13) might not be significant. If the definition of the interest rate is based on the criteria described in the previous paragraph, the use of the time-dependent solution (equation (12)) becomes indisputable.

In addition to the considerations above, the location of the optimal point can also be affected by design time considerations. In order to illustrate this point, consider a facility with a set of given functions \( B(p) \), \( D(p) \), \( C(p) \) and a fixed interest rate. The change of \( Z(p, T) \) for different values of \( T \) can be observed in figure 3. As the design period grows, the optimal value converges to the optimum obtained using \( Z_R(p) \) (i.e. \( p_{opt} = 68.821 \)) (equation (13)). In other words, the optimal solution provided by \( Z_R(p) \) corresponds to the maximum optimum and may over-estimate an optimum solution based on the actual life expectancy of the facility. This raises the question: should large infrastructure systems be designed for a specific lifetime, or should decisions assume an ‘eternal’ renewal and updating process? A case that illustrates this type of inconsistency is the construction of highways,
which have been implicitly designed to exist for extremely long time periods, while the pavement structures are designed for a finite operation period (e.g. 15–20 years).

4. Reliability profile

In previous sections the case where the behaviour of the facility with time followed a renewal model was
discussed. In this model, it was assumed that the facility operates under optimal conditions until the failure occurs, then it is repaired and taken back to its original standards and operation continued as it was originally conceived. Nevertheless, most structures, and infrastructure systems in particular, do not keep the same reliability level throughout all time. On the contrary, they suffer a process of degradation, which may eventually lead to failure even before an extreme event.

The reliability profile is a function that describes the change of the facility’s reliability with time. Its importance is based on the fact that as the performance (reliability) gets closer to a prescribed target failure probability, a maintenance action is required. Frangopol et al. (1997) distinguish between actions and events that take place during the serviceable life of a facility. Actions can be defined as scheduled interventions on the structures (e.g. partial and full maintenance) and events that are unexpected incidents that may cause the failure (e.g. deterioration and failure caused by earthquakes). In the evaluation of the reliability profile, is necessary to quantify the change of the reliability with time. This is commonly reflected in a decrement of the safety due to ageing, environmental factors, or chemical actions. Several models have been proposed to describe the reliability profile. Some assume that deterioration is linear with time and other describe it by using a probability distribution. For instance, the reliability profile has been described by Yang et al. (2005) in terms of a survival function as:

\[ P_s = S(t) = e^{-\lambda (t-t_0)^k}, \]  

(14)

where \( \lambda \) is the failure rate, \( t \) is the time, \( t_0 \) is the time at the beginning of the process and \( k \) is a shape factor. Then, failure probability can be computed as \( p_f = 1 - P_s \). As suggested by the authors, a small adjustment is required to ensure that \( p_f \neq 0 \) at time \( t = 0 \). Higher values of the shape factor \( k \) result in longer time periods to reach the target failure probability.

Another classical model has been proposed by Mori and Ellingwood (1994) where the degradation function of structural components is based on individual intensities. Their work focused mainly on concrete structures. Thus, the variation of the structural resistance with time can be computed as:

\[ R(t) = G(t) \cdot R_0, \]  

(15)

where \( R_0 \) is the initial resistance; and \( G(t) \) is the degradation function, defined as:

\[ G(t) = 1 - \max_{j} X_j(t), \]  

(16)

where \( j \) represents the number of locations where damage has begun, and \( X_j \) is the damage intensity at location \( j \). \( X_j(t) \) is described by the following function:

\[ X_j(t) = \begin{cases} 0 & 0 \leq t \leq T_y \\ C_j(t - T_y) & t > T_y \end{cases}, \]  

(17)

in which \( C_j \) is the damage growth rate at location \( j \), \( z \) is a control parameter that has to be calibrated to adjust the model, and \( T_y \) are the times at which deterioration initiates. Initialization time can be modelled in several ways, e.g. by assuming that it follows a Poisson process or by simulating the facility’s degradation process. Mori and Ellingwood (1994) found that the variation of the degradation function with time could be described by its first two moments:

\[ E[G(t)] = 1 - \int_0^1 [1 - F_{X_{\max}}(x; t)] dx, \]  

(18)

\[ Var[G(t)] = \int_0^1 2x[1 - F_{X_{\max}}(x; t)] dx - E[G(t)]^2, \]  

(19)

where \( F_{X_{\max}}(x; t) \) is the cumulative distribution function of the maximum damage intensity. This is described by the individual cumulative distribution function of the \( n \) locations where damage has begun. The cumulative distribution function depends on the statistical parameters that describe the damage growth and the initialization times. An important result is that values of variance were so small that the degradation function can be treated using only first-order statistics. For more information, refer to Mori and Ellingwood (1994).

5. Maintenance within the life cycle analysis

The scheduled maintenance of a facility can be described by a set of intervention times, \( t_i \). This set can also contain the value of the change in reliability associated with each intervention, i.e. \( \Delta \beta_i \). Thus, the maintenance policy can be defined as:

\[ \Omega = \{ \{t_1, t_2, \ldots, t_m\} \cup \{\Delta \beta_1, \Delta \beta_2, \ldots, \Delta \beta_m\} \}, \]  

(20)

with \( m \) maintenances during the design period. When maintenance is included into the life cycle cost analysis, a new term has to be added to equation (1). Then, this equation is modified to:

\[ Z(p, \Omega) = B - C(p) - D(p, T) - M(p, \Omega), \]  

(21)

where \( M(p, \Omega) \) is the maintenance cost. Yang et al. (2005) suggested that preventive maintenance can be divided into proactive and reactive. The former is carried out before the prescribed threshold value has been reached, while the
reactive part is performed after this limit has been exceeded.
The first approach postpones the time to the expected intervention, while the latter is reflected in an updating of the structural condition. In the model proposed in this paper, a plan for proactive maintenances is optimized, while the expected cost of reactive maintenance due to a failure is considered as a failure within the cost-optimization function.
Frangopol (1997) proposed a model for evaluating maintenance costs of bridges based on comparing the resistant moment before and after the intervention to predict the damage of the bridge and the effectiveness of the maintenance. This model can be modified and extended to include the reliability expressed in a broader sense. According to this model, the intervention has an effect on the facility that can be described as:
\[
e_{\text{int}} = \frac{R_a - R_b}{R_0}
\]
(22)
where \(R_0\) is the original reliability, \(R_a\) is the value of reliability after intervention and \(R_b\) is the value before intervention. This effect can also be formulated as a function of the failure probability as:
\[
e_{\text{int}} = \frac{P_{fb} - P_{fa}}{1 - P_{f0}}.
\]
(23)
A special case can be defined when the failure probability after intervention is the same as the original failure probability. This can be seen as a complete effective maintenance. In this case the effect can be simplified to:
\[
e_{\text{int}}^* = 1 - \frac{1 - P_{fb}}{1 - P_{f0}}.
\]
(24)

The effect described up until now directly affects the replacement cost of the facility. Notice that when \(P_{fb} = 1\), \(e_{\text{int}}^* = 1\) and the maintenance cost becomes the replacement cost. If it is assumed that the replacement cost is equal to the basic maintenance cost plus the construction cost \(C(p)\), the maintenance expenses can be calculated as:
\[
M(p, \Omega) = \sum_{i=1}^{m} [(M_0 + C(p)) \times (e_{\text{int},i})^k \delta(t_i) + H],
\]
(25)
where \(M_0\) is the basic maintenance cost; \(k\) is a model parameter (assumed to be 0.5, Frangopol et al. 1997 and Radojicic 2002); \(\delta(t)\) is the capitalization function defined by equation (3); and, as mentioned previously, \(H\) accounts for costs such as loss of opportunity, demolition, removal, etc. In equation (25), \(e_{\text{int}}\) can be replaced by \(e_{\text{int}}^*\) when appropriate.

6. Numerical example
In order to illustrate the concepts described above, the case of a simple supported reinforced concrete beam subjected to infrequent overloads and located in a very aggressive environment will be studied. First of all, the model of corrosion due to chloride penetration, which will be taken as the degradation model for the structure, will be briefly described. Then, the optimization will be carried out in order to obtain the appropriate beam dimensions, the amount of steel reinforcement required and the proactive preventive maintenance program.

6.1 Reinforcement corrosion due to chloride penetration model
One of the most common causes of deterioration of concrete structures is the penetration of chlorides or other chemical substances. Reinforced concrete elements located in the presence of a high concentration of chemical substances (e.g. chlorides) may be subject to steel corrosion. The consequences of corrosion can be observed by a reduction in the steel bars’ effective area. This topic has been studied extensively and although there are many different models in the literature, the parameters that describe the physical phenomenon are rather similar. The reduction of the steel bars’ diameter can be described by:
\[
d(t) = \begin{cases} 
d_0 & 0 \leq t \leq T_f \\
 d_0 - v \times (t - T_f) & t > T_f,
\end{cases}
\]
(26)
where \(d_0\) is the initial diameter of the bar, \(T_f\) is the time for which the corrosion begins and \(v\) is the steel corrosion rate. The time at which corrosion starts is described by the concentration in the surface of the beam \(C_0\), a critical concentration \(C_{cr}\), a diffusion coefficient \(D\), and the covering width \(c\). Critical concentration is defined as the concentration when the chloride starts penetrating the concrete. Thus, the initialization time is defined by Hong (2000) as:
\[
T_i = \begin{cases} 
\frac{1}{2D} \left[ \Phi^{-1} \left( \frac{c}{2D} \right) \right] & C_{cr} \leq C_0, \\
\infty & C_{cr} > C_0
\end{cases}
\]
(27)
where, \(\Phi^{-1}(\cdot)\) is the cumulative normal distribution. Lounis and Amleh (2003) developed a similar approximation for the initialization time function, but defined it in terms of the error function:
\[
T_i = \begin{cases} 
\frac{1}{4D} \left[ \text{erf}^{-1} \left( \frac{c}{2D} \right) \right]^2 & C_{cr} \leq C_0, \\
\infty & C_{cr} > C_0
\end{cases}
\]
(28)
where

\[ \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-y^2) dy. \] (29)

The random variables \( D, C_{cr}, C_0 \) and \( v \) can be found experimentally, or some typical values can be found in the literature (see table 1).

### 6.2 Optimization

#### 6.2.1 Problem formulation.
Consider a simply supported beam for which the parameters and the maintenance programme will be optimized so that the economic benefit is maximized. The loading consists of its own weight and two random overloads that are applied at 1/3 and 2/3 of the span (see figure 4). For the purpose of this example, the reliability problem was formulated in terms of the limit state equation defined by the bending moment in the centre of the span:

\[ M_R - M_A = 0, \] (30)

\[ A_S f_y \left( 1 - 0.59 \frac{A_S f_y}{b \cdot d \cdot f_c} \right) - \left( \frac{F L^3}{3} + 0.0024 b \cdot d \cdot L^2 \right) = 0, \] (31)

where \( b \) and \( d \) are the width and the depth of the beam, \( L \) is the length of the span, \( A_S \) is the area of steel reinforcement, \( f_y \) is the yield strength of the steel bars, \( f_c \) is the concrete strength and \( F \) is the magnitude of the loading.

#### 6.2.2 Parameters for optimization.
In order to formulate the optimization problem, the parameters were organized into five groups:

- Economical (see table 2)
- Material resistance (see table 3)
- Dimensions (see table 4)
- Applied stochastic loading (see table 5)
- Degradation parameters (see table 6)

Based on these assumptions, the objective function becomes \( Z(T, b, d, q, m, t_1, t_2, \ldots, t_m) \):

**Maximize:** \( Z(T, b, d, q, m, t_1, t_2, \ldots, t_m) \)

s.t. \( 0.25 \leq b \leq 0.60; 0.30 \leq d \leq 1.0, \)

\( P_f(p)_{i=0} \leq 0.05; P_f(p)_{i>0} \leq 0.2 \)

\( t_{i+1} - t_i \geq t_{i\text{min}} \) (32)

Table 1. Typical values for modelling corrosion.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Mean value</th>
<th>Coefficient of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D )</td>
<td>Lognormal</td>
<td>51–63 mm²/y</td>
<td>0.40–0.75</td>
</tr>
<tr>
<td>( C_{cr} )</td>
<td>Lognormal</td>
<td>0.5–4% by cement weight</td>
<td>0.10–0.20</td>
</tr>
<tr>
<td>( C_0 )</td>
<td>Lognormal</td>
<td>1.2–1.4% by cement weight</td>
<td>0.40–0.50</td>
</tr>
<tr>
<td>( v )</td>
<td>Normal</td>
<td>0.075 mm/y</td>
<td>0.30</td>
</tr>
</tbody>
</table>


Figure 4. Numerical example: simple supported beam in an aggressive environment subject to two extreme infrequent loads.
between maintenances, in this case it was assumed to be 5 years. Restrictions on the dimensions and the steel ratio correspond to the requirements and specifications of the code of practice and specific constructive considerations (e.g. requirements of space between bars). The initial value of failure probability is restricted to 0.05 and the minimum probability limit to a threshold of 0.2. This threshold of the probability of failure illustrates the case of large infrastructure systems; however, it has to be determined carefully for every component and for the specific nature of the system. The probability profile was calculated according to section 6.1, with parameters as shown in table 6. In order to simplify the optimization only one type of bar diameter of steel reinforcement was considered.

### Table 2. Economic parameters for optimization.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Cost</td>
<td>US$/kg</td>
<td>1.4</td>
</tr>
<tr>
<td>Concrete Cost</td>
<td>US$/m³</td>
<td>103.3</td>
</tr>
<tr>
<td>Annual Benefit (present)</td>
<td>$b₀</td>
<td>44</td>
</tr>
<tr>
<td>Basic Maintenance Cost</td>
<td>$M₀</td>
<td>4.4</td>
</tr>
<tr>
<td>Cost Associated to the Failure</td>
<td>$H</td>
<td>6458</td>
</tr>
<tr>
<td>Discount Rate</td>
<td>γ</td>
<td>6%</td>
</tr>
</tbody>
</table>

### Table 3. First two moments of the steel and concrete distributions.

<table>
<thead>
<tr>
<th>Material resistance</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Resistance (mean)</td>
<td>MPa</td>
<td>420</td>
</tr>
<tr>
<td>Steel Resistance (Std. Dev.)</td>
<td>MPa</td>
<td>42</td>
</tr>
<tr>
<td>Concrete Resistance (mean)</td>
<td>MPa</td>
<td>28</td>
</tr>
<tr>
<td>Concrete Resistance (Std. Dev.)</td>
<td>MPa</td>
<td>3.6</td>
</tr>
</tbody>
</table>

Both concrete and steel resistance were assumed to be normally distributed.

### Table 4. Geometric parameters for optimization.

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Length</td>
<td>L</td>
<td>500</td>
</tr>
<tr>
<td>Beam Width (minimum value)</td>
<td>cm</td>
<td>25</td>
</tr>
<tr>
<td>Beam Width (maximum value)</td>
<td>cm</td>
<td>60</td>
</tr>
<tr>
<td>Beam Depth (minimum value)</td>
<td>cm</td>
<td>30</td>
</tr>
<tr>
<td>Beam Depth (maximum value)</td>
<td>cm</td>
<td>100</td>
</tr>
<tr>
<td>Max. Diam. of Reinf. Steel</td>
<td>cm</td>
<td>1.91</td>
</tr>
</tbody>
</table>

### Table 5. Stochastic loading parameters for optimization.

<table>
<thead>
<tr>
<th>Force characteristics</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force Magnitude (mean)</td>
<td>kN</td>
<td>50</td>
</tr>
<tr>
<td>Force Magnitude (Std. Dev.)</td>
<td>kN</td>
<td>5</td>
</tr>
</tbody>
</table>

An occurrence rate of disturbances of $\lambda=0.04$ was assumed.

### Table 6. Parameters of the degradation model.

<table>
<thead>
<tr>
<th>Degradation parameters</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initialization Time</td>
<td>y</td>
<td>0</td>
</tr>
<tr>
<td>Corrosion Rate Steel (mean)</td>
<td>cm/y</td>
<td>0.0075</td>
</tr>
<tr>
<td>Corrosion Rate Steel (Std. Dev.)</td>
<td>cm/y</td>
<td>0.0023</td>
</tr>
</tbody>
</table>

The corrosion rate was assumed normally distributed.

### 6.3. Results

The result of the optimization provides the structural dimensions (i.e. $b$, $d$, $diam$, $q$) and the maintenance programme (i.e. number and time of preventive interventions) that maximizes the objective function. It was assumed that any intervention takes the structure to its

### Table 7. Results for $T = 25$. 0* Without degradation. 0** With degradation.

<table>
<thead>
<tr>
<th>Policy</th>
<th>$Z(p)$</th>
<th>$b$</th>
<th>$d$</th>
<th>diam</th>
<th>No.</th>
<th>$T₁$</th>
<th>$T₂$</th>
<th>$T₃$</th>
<th>$T₄$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0*</td>
<td>440.63</td>
<td>25</td>
<td>45</td>
<td>1.59</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0**</td>
<td>413.36</td>
<td>25</td>
<td>55</td>
<td>1.91</td>
<td>3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>385.02</td>
<td>25</td>
<td>50</td>
<td>1.91</td>
<td>3</td>
<td>20</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>346.48</td>
<td>25</td>
<td>40</td>
<td>1.91</td>
<td>3</td>
<td>12</td>
<td>20</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>288.06</td>
<td>25</td>
<td>40</td>
<td>1.91</td>
<td>3</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>200.88</td>
<td>25</td>
<td>40</td>
<td>1.91</td>
<td>3</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
</tr>
</tbody>
</table>

### Table 8. Results for $T = 40$. 0* Without degradation. 0** With degradation.

<table>
<thead>
<tr>
<th>Policy</th>
<th>$Z(p)$</th>
<th>$b$</th>
<th>$d$</th>
<th>diam</th>
<th>No.</th>
<th>$T₁$</th>
<th>$T₂$</th>
<th>$T₃$</th>
<th>$T₄$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0*</td>
<td>536.31</td>
<td>25</td>
<td>40</td>
<td>1.91</td>
<td>3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0**</td>
<td>475.44</td>
<td>25</td>
<td>60</td>
<td>1.91</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>483.92</td>
<td>25</td>
<td>55</td>
<td>1.91</td>
<td>3</td>
<td>25</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>479.33</td>
<td>25</td>
<td>45</td>
<td>1.91</td>
<td>3</td>
<td>22</td>
<td>44</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>458.77</td>
<td>25</td>
<td>45</td>
<td>1.91</td>
<td>3</td>
<td>22</td>
<td>40</td>
<td>45</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>431.01</td>
<td>25</td>
<td>45</td>
<td>1.91</td>
<td>3</td>
<td>22</td>
<td>35</td>
<td>40</td>
<td>45</td>
</tr>
</tbody>
</table>

### Table 9. Results for $T = 50$. 0* Without degradation. 0** With degradation.

<table>
<thead>
<tr>
<th>Policy</th>
<th>$Z(p)$</th>
<th>$b$</th>
<th>$d$</th>
<th>diam</th>
<th>No.</th>
<th>$T₁$</th>
<th>$T₂$</th>
<th>$T₃$</th>
<th>$T₄$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0*</td>
<td>566.30</td>
<td>25</td>
<td>40</td>
<td>1.91</td>
<td>3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0**</td>
<td>471.70</td>
<td>25</td>
<td>90</td>
<td>1.91</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>514.36</td>
<td>25</td>
<td>55</td>
<td>1.91</td>
<td>3</td>
<td>25</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>519.07</td>
<td>25</td>
<td>45</td>
<td>1.91</td>
<td>3</td>
<td>22</td>
<td>44</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>508.33</td>
<td>25</td>
<td>45</td>
<td>1.91</td>
<td>3</td>
<td>22</td>
<td>40</td>
<td>45</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>493.10</td>
<td>25</td>
<td>45</td>
<td>1.91</td>
<td>3</td>
<td>22</td>
<td>35</td>
<td>40</td>
<td>45</td>
</tr>
</tbody>
</table>
original standards. Three lifetime cases were considered: 25, 40 and 50 years, and six possible intervention programs: two cases of no intervention and four possible interventions. Furthermore, for the non-maintenance policy, two special conditions were considered: with and without deterioration. The case indicated as 0* corresponds to the optimum design without degradation (a constant reliability profile); while the case 0** includes a structural degradation process taking into account a reliability threshold established in the restrictions of equation (32).

The optimum structural dimensions and the corresponding intervention times for every maintenance policy are presented in tables 7, 8 and 9. It can be observed that in all three cases there is a difference between the cases named 0* and 0**. For the case of no degradation (i.e. 0*), the optimum dimensions are constant independent of the structure’s design life. Nevertheless, if it is assumed that the structure degrades with time, changes in the structural dimensions are significant for all three life spans considered. This is justified by the fact that when there is not a maintenance policy, the structure has to delay any possible failure until the end of its life and that implies a more robust and therefore more expensive design. Then, when comparing the no maintenance policy designs for 25, 40 and 50 years, it is observed that as the expected life of the structure grows, the section of the beam increases ($T=25; (b \times h)_{0*} = 25 \times 55; T=40; (b \times h)_{0*} = 25 \times 60; T=50; (b \times h)_{0*} = 25 \times 90$). It is also observed that, with the exception of the case of one maintenance, the beam’s cross section keeps the same dimensions. This is caused by the fact that, in this particular case, the dominant factor is the intervention cost and not either the initial cost or the

Figure 5. Reliability profile defined by the optimum maintenance programme for the three lifetimes considered.
expected losses. Several sensitivity analyses have shown that an intensive maintenance programme reduces the cross section but it is independent of the life span. Finally, it is also observed that for all three lifetime cases and maintenance policies, the optimum steel reinforcement remains constant; the explanation of this is the same as that of the analysis of the cross section.

With regards to the maintenance program, the optimum number of interventions grows when the lifetime of the structure is larger. For the case of 25 years, the optimum result is 0 maintenances; for 40 years the objective function is maximized when one maintenance is considered (25 years); and for the case of 50 years, the optimum number of maintenances is two (22 and 44 years). The optimum number of interventions is highly dependant on the associated cost. It was observed that as the number of interventions grow, the value of the objective function decreases. Note also that interventions are not defined at regular time intervals. It was observed that for larger lifetime periods the number of required interventions that maximize $Z(p)$ increased, but they were biased towards the end of the structural life, thus reducing the impact on the objective function. The probability profiles for different maintenance policies and for all lifetime cases are shown in figure 5. It can also be observed that the profile for ‘No Maintenance without degradation (i.e. 0*)’ reaches

![Figure 6. Objective function value for different numbers of maintenances: $Z(p, t = 25)$, $Z(p, t = 40)$ and $Z(p, t = 50)$.](image)

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a threshold value of 0.2 at 15 years, meanwhile the case of ‘No Maintenance with degradation (i.e. 0**)’ does not even get to this value. This behaviour takes place no matter which design life time is considered.

As shown in tables 7, 8 and 9, the value of the objective function is larger when the no maintenance policy without degradation is considered. If the optimization is restricted to a set of periodic interventions (i.e. performed at regular intervention times), only vector \( p \) is optimized. Figure 6 shows a comparison between a policy of periodic interventions compared with the optimal solution found through optimization. It can be observed that periodic interventions are not always the best solution; in fact, in most cases, non-periodic interventions lead to a higher value of the objective function. In all three lifetime cases it can be noticed that as long as the number of maintenances grows, the difference between the optimum solution and the solution with periodic interventions also increases. This increment is higher as the design time increases.

7. Conclusions

Future engineering projects have to be looked at from a new perspective within which their life cycle is taken into account. Since the decision on investing in any project is usually made within an economic framework, appropriate models for costing the construction, operation, maintenance and disposal should be designed. These models must also take into account aspects that are relevant to the decision, but not easily quantifiable, such as the environmental impact and the intergenerational responsibility.

This paper presents a model for optimizing both the mechanical parameters and the maintenance programme of a new project. It encompasses structural deterioration, the possibility of infrequent events, preventive maintenance and possible reconstruction after failure. The model was illustrated with an example of a RC beam for which the dimensions of the cross section, the steel reinforcement and the maintenance programme were optimized. The analysis was carried out for three structural life spans and different maintenance policies. It was observed that as the life span grows, the number of maintenances grows, and are biased towards the end of the structural life in order to reduce the impact on the objective function. When no maintenance is possible, the cross section becomes larger as the lifetime grows because the failure is not allowed. On the whole, the optimum solution provides a balance between safety considerations, the structure’s life and economic aspects.

Further work has to be carried out on cost-based structural optimization to cover aspects that have shown to be critical for decision making, such as the definition of the interest rate and other socioeconomic considerations. Mechanical and computational issues which still require some work also have to be treated, but defining the appropriate socioeconomic context is becoming the keystone for future developments and practical implementation.

References


Rackwitz, R., Risk control and optimization for structural facilities, in 20th IFIP TC7 Conference on System Modelling Optimization, July 23 – 27, 2001, Trier, Germany.


