Comparing target spectral design acceleration values by using different acceptability criteria

Mauricio Sanchez-Silva *, Orlando Arroyo

Civil and Environmental Engineering, Universidad de Los Andes, Carrera 1 No. 18A-70 Edificio W, Piso 3, Bogotá, Colombia

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Abstract

A central issue in earthquake engineering design is the treatment of ground motion uncertainty and the non-linear structural response for defining the design acceleration. This paper first proposes a probabilistic design spectrum which includes as input, in addition to common design parameters, the probability that the design acceleration is exceeded. Furthermore, it presents several alternatives for defining the bounds of the ALARP region to define target reliability values. In order to define acceptable probabilities of exceedence several alternatives have been considered and compared. Finally, the importance and the impact of selecting target reliabilities is demonstrated for the case of low income housing developments in Colombia.

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1. Introduction

At present, structural design is based on rational and widely accepted mechanical models despite the fact that there are still many sources of uncertainty both inherently random and epistemic in nature. Structural loads and strengths are unpredictable, databases are limited and
modeling of performance limit states cannot be carried out accurately [8]. Although these aspects are common to all countries, the consequences of safety-decisions for a society in terms of development and quality of life have been kept aside. This paper presents and compares several strategies for deciding on appropriate seismic structural design criteria.

2. Earthquake engineering design

2.1. General aspects

Earthquake design is based on well-known mechanical models over which there is little dispute. Safety requirements are based on statistical data, collected mainly in countries where it is available (e.g. US), and on widely accepted earthquake intensity exceedence criteria. Safety requirements specified in modern codes of practice describe the expected performance of structural and non-structural components, subjected to earthquake loading, under the following precepts: (1) no damage to either structural or non-structural components during minor shaking; (2) limited non-structural damage, but no damage to structural components during moderate shaking; and (3) structural and non-structural damage during severe shaking, total building collapse should be prevented [1].

Since a wide range of structural performance requirements may be defined by building owners, FEMA [23] defines four basic structural performance levels, which are expressed in terms of probability of exceedence: (1) immediate occupancy (50% in 50 years or 72 year return period), (2) life safety (20% in 50 years or 225 year return period), (3) collapse prevention (10% in 50 years or 475 year return period) and (4) special cases (cases not considered) (2% in 50 years or 2475 year return period); and two intermediate structural performance ranges: (1) damage control range, and (2) limited safety range. Structural seismic performance goals are commonly set as: (1) $p_f = 1 \times 10^{-3}$ for maintaining occupant safety; (2) $p_f = 5 \times 10^{-4}$ for maintaining occupant safety and continuing operation with minimal interruption; (3) $p_f = 1 \times 10^{-4}$ for maintaining occupant safety and continuing operation with minimal interruption – hazard confinement; and (4) $p_f = 1 \times 10^{-5}$ for maintaining occupant safety, hazard confinement, and excessive damage [1].

In summary, a seismic performance goal for general use facilities is the prevention of major structural damage, or facility collapse that would endanger the occupants. In most codes of practice, structures are designed for earthquake intensity with a probability of being exceeded by 10% in 50 years. Maxima design criteria to avoid collapse refer to earthquake events with return periods between 1000 and 1500 years, while for service conditions return periods are within the range of 20–50 years. Wen [22] argues that even though these values are widely accepted, to strictly enforce reliability performance goals, target probabilities need to be set directly for the limit states rather than for the design earthquake.

2.2. Probabilistic design response spectrum

Earthquake loading is defined in terms of the design response spectrum. It is used to obtain the design spectral acceleration based on the fundamental vibration period of the building [15, Section 9.5.3.2.1], the occupancy important factor [15, Section 9.1.4], and the site coefficients [15, Section
Despite the fact that the design spectrum has been widely used in practice, research during the last years has made clear that simple representation of ground motion by static design coefficients does not capture the diversity of engineered buildings. The ATC-34 [1] states that “recent initiatives to develop performance-based seismic design procedures are questioning the applicability of conventional spectra and peak ground acceleration representations. They further emphasize that meeting the needs of regulators and the designers and owners of individual buildings requires careful rethinking”.

Based on the need to reformulate earthquake design criteria, the first aspect that becomes evident is that the design spectrum does not consider failure probability as an input parameter. This is clearly an important deficiency since it is at the bottom of the analysis and is the basis for performance based design. Therefore, it is suggested that in order to include the failure probability, a new dimension describing the probability that the design acceleration is exceeded should be added to the current design spectrum. Then, the probabilistic design response spectrum can be defined in terms of failure probability as

\[
S_a(T, S, I, p_f) = \frac{K}{R} A_g(p_f),
\]

where \(A_g(p_f)\) is the peak ground acceleration as function of the failure probability, \(K\) is a factor that depends upon the region of the spectrum (e.g. \(K = 2.5I\) for the intermediate region), and \(R\) is the response modification factor. Note that what is called failure probability is not related to the failure of a particular element or the formation of a mechanism of collapse; it describes the probability that the design acceleration is exceeded.

### 2.3. Earthquake probabilistic model for computing \(A_g(p_f)\)

In order to compute \(A_g(p_f)\), the following limit state function is considered:

\[
g(S_a, A_g) = S_a - K \varepsilon A_g = 0,
\]

where \(S_a\) corresponds to the design acceleration and \(A_g\) to the peak ground acceleration at the base. Constant \(K\) is an amplification factor which is a function of \(I\), \(S\) and \(T\) and \(\varepsilon\) is a variable that accounts for the variability of the response spectral acceleration and the uncertainty in the attenuation law (\(\mu_S = 1\), \(\sigma_S = 0.8\)). \(S_a\) was assumed to be distributed Lognormal with a coefficient of variation of 20%, although several authors [5] have estimated the uncertainty in the capacity side to be 40% or higher; this is significant in itself but small compared with the uncertainty of seismic loading.

The distribution for \(A_g\) depends upon the seismic model considered. For convenience and in agreement with current practice, the peak ground acceleration is a function of earthquake magnitude \(M\) and epicentral distance \(R\). Magnitude was fitted by an extreme value distribution type III (for maxima) [20]. The upper bound is defined by historical data of earthquake events and by the regional geological characteristics. In addition, only events with magnitudes which may cause significant damage are taken into account; i.e. \(m \geq M_{\text{min}} = 4.0\) [18]. Attenuation laws relating peak ground acceleration with magnitude and epicentral distance have the following general form:

\[
a = h(m, r) = b_1(r)e^{b_2m},
\]
where \( a \) is the acceleration at the site of interest described later in this paper, \( b_2 = 0.573 \), \( m \) is the magnitude and \( b_1(r) \) is a function of distance describing the energy dissipation process. For the attenuation law in Eq. (3) a coefficient of variation of about 0.6 is reported. Based on the attenuation law for the seismic conditions of California [11], which also may be applicable to the region considered later in this study, the derived conditional density distribution function for the acceleration is given by [20]

\[
\begin{align*}
  f_A(a,r) & = \frac{\alpha}{M_m - u} \left( \frac{M_u - h^{-1}(a,r)}{M_m - u} \right)^{w - 1} \exp \left[ - \left( \frac{M_u - h^{-1}(a,r)}{M_m - u} \right)^w \right] \frac{dh^{-1}(a,r)}{da}, \\
  \text{with } M_m < h^{-1}(a,r) = (1/b_2) \ln(a/b_1(r)) < M_u.
\end{align*}
\]

In order to obtain the unconditional density function for the acceleration it is necessary to integrate over the area within which the analysis is performed. This is assumed to be circular around the point of interest with \( R_{\text{max}} = 200 \) km. The probability density function of the distance \( R \) has been considered as uniform with a value of \( f_R(r) = 2r/R_{\text{max}}^2 \) [20]. More elaborated models can be developed if the location and earthquake pattern of the seismic sources are clearly identified, but simple model is fully sufficient for the purpose of this study. Therefore, the on-site density of acceleration is

\[
  f_A(a) = \int_0^{R_{\text{max}}} f_A(a,r)f_R(r) \, dr.
\]

The mean and the standard deviation for the acceleration are 0.075 and 0.116 m/s\(^2\), respectively; implying a coefficient of variation of 155\%. According to the attenuation law, the maximum and minimum acceleration expected in site are 9.44 m/s\(^2\) \((R = 0, M = M_u)\) and 0.014 m/s\(^2\) \((R = 200 \) km, \( M = 4)\), respectively. The seismic data used are characteristic to an area with medium to high seismicity.

The probability density of the peak ground acceleration can be obtained from standard probabilistic techniques. Nevertheless, regardless of the probability model of the ground motion, what is important is computing the probability of failure based on the limit state equation (Eq. (2)). The probability that this limit state is violated is calculated for different mean values of \( S_a \) using well-known reliability models FORM/SORM. It is then possible to find the curve described in Fig. 1 in which the 3D response spectrum is shown. It is important to stress that instead of probability, the new axis may be described in terms of return period so that it can compared with classic ways of describing performance levels (Section 2.1). Besides, note that the design spectrum, specified in the code of practice, is a particular case.

It can also be observed that, as expected, the design acceleration decays as the failure probability increases. In other words, it means that if lower design acceleration values are selected, the probability that the ground motion exceeds it increases. For instance, a structure designed for an acceleration of \( S_a = 2.0 \) m/s\(^2\) and \( K/R = 0.55 \) has a probability of \( p_f = 7.67 \times 10^{-5} \) that the design acceleration is exceeded; while for \( S_a = 1.0 \) m/s\(^2\) the probability is \( p_f = 1.1 \times 10^{-3} \). What is interesting about this spectrum is that there is not a pre-set failure probability as in the code of practice; on the contrary, it becomes an additional design parameter. Thus, the determination of the spectral design acceleration requires, in addition, \( T, S, I \) and \( R \) factors, the selection of a probability of failure. A discussion on the selection of failure probability is presented in next section.
3. Risk acceptability

The definition of acceptable level of risk has always been a key issue in structural design. In reliability terms, this is related to the decision about whether the probability of a limit state violation is acceptable or not; or on selecting a failure probability for which the structure is safe enough. However, the decision about acceptance also has to include an assessment of the consequences of failure and the context within which an unfavorable event might happen. Among the most common criteria for making decisions about risk acceptance are the comparison between the calculated probability of structural failure with other risk in society, the definition of the ALARP (As Low As Reasonably Possible) region, the calibration at past and present practice, and the cost-benefit analysis [20].

Defining acceptable risk by comparing death rates of different activities within a society may be misleading. Acceptability of risks varies with age, gender, socioeconomic conditions, level of education, cultural background, available information, media influence, physiological aspects, and so forth. Special attention should be given to the differentiation between individual and collective risks. An individual acts with respect to his/her needs, preferences and lifestyle. Thus, risk acceptance depends on the degree to which the risk is incurred voluntarily. On the other hand, collective (public) risk is of concern for the government, or the operator of a technical facility, who acts on behalf of society as a whole and is not concerned with the individual’s safety. The ALARP approach defines a region of acceptable values of probability of failure in a plot of the occurrence
probability of adverse events versus their consequences. Although this approach might be appealing, there are significant difficulties in its interpretation, openness of decision processes, morality of actions and comparability between facilities [13]. Calibration of acceptable levels of risk at past and present practice has also been used for defining target reliabilities. It is tacitly assumed that this practice is optimal although this is not at all obvious. Despite the fact that its development is based on trial and error, this calibration cannot give totally wrong numbers because of their long history; nevertheless there is great variation of in reliability levels. Finally, a reliability oriented cost-benefit analysis considers that technical facilities should be economically optimal. This approach has been recently updated by including the life quality index (LQI) as proposed by Nathwani et al. [14], leading to the conclusion that risk acceptability from the public perspective is essentially a matter of efficient investment into life saving measures [20].

3.1. Defining the bounds for the ALARP region

Many criteria for defining target reliabilities for earthquake design have been discussed in the literature, and the selection of specific values depends upon many particular considerations about the project, the region and the socioeconomic context where it will be built. In this section, two criteria for defining the bounds of the ALARP region will be described.

3.1.1. Performance design criteria

In structural performance design, target failure probabilities for different operation and safety conditions have been agreed on within the earthquake community based on experience (Fig. 2). Both operation and safety depend on the structural behavior, and especially of the inelastic response. Therefore, the upper and lower limits of the ALARP region are somehow defined in most codes of practice by a response modification coefficient, $R$. The factor $R$ is used to reduce the elastic spectrum to an allowable-stress design spectrum, whereby it is assumed that a lateral-force-resisting system will undergo significant inelastic behavior during the design earthquake. The factor $R$ depends upon the type of lateral-force-resisting system and ranges between 6 and 12 for most

<table>
<thead>
<tr>
<th>Earthquake design level (Prob. of exceedence)</th>
<th>Fully Operational</th>
<th>Basic Operation</th>
<th>Life Safety</th>
<th>Collapse Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>50% in 50 years (T=72 years)</td>
<td></td>
<td></td>
<td></td>
<td>$P_{exc} = 1.38 \times 10^{-2}$</td>
</tr>
<tr>
<td>20% in 50 years (T=225 years)</td>
<td></td>
<td></td>
<td>Unacceptable performance</td>
<td>$P_{exc} = 4.44 \times 10^{-3}$</td>
</tr>
<tr>
<td>10% in 50 years (T=475 years)</td>
<td></td>
<td>ALARP Region</td>
<td></td>
<td>$P_{exc} = 2.10 \times 10^{-3}$</td>
</tr>
<tr>
<td>2% in 50 years (T=2475 years)</td>
<td>Over-design</td>
<td></td>
<td></td>
<td>$P_{exc} = 4.04 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Fig. 2. Definition of acceptable risk levels for common design performance levels.
conventional building structures. The \( R \) factor is a global measure intended to reflect reductions in design force justified on the basis of risk assessment, economics and non-linear behavior [1]. A draft formulation for \( R \) proposed in the ATC-19 [2] specifically addresses several factors that influence the seismic response of the building during severe shaking; a detailed description of these factors is beyond the scope of this paper but can be found in [1].

Similarly to factor \( R \), for rehabilitation purposes FEMA [23] defines the factor \( m \) as the “component or element demand modifier to account for expected ductility associated with this action at the selected Structural Performance level”. Therefore, the expected strength of the structure at the deformation level under consideration for deformation-controlled actions, \( Q_{CE} \), multiplied by \( m \) should be higher than the deformation controlled design action due to gravity and earthquake loads, \( Q_{UD} \), i.e. \( m Q_{CE} \geq Q_{UD} \). Typical values of \( m \) for structural performance life safety are: beams controlled by flexure: \( m = 2–6 \); beams controlled by shear: \( m = 1.5–3 \); columns controlled by flexure: \( m = 1–4 \); columns controlled by shear: \( m = 1.5–2 \); and shear walls: \( m = 1.75–6 \). Note that while the \( R \) factor is a global measure, \( m \) factors are related to individual members.

It is well known in practice that design acceleration values higher than those defined by the spectrum with a response reduction factor \( R = 1 \) lead to over-designs, which are inefficient (upper limit of the ALARP region). However, defining the lower part of the ALARP region is a difficult task. Imbedded in the design philosophy established in codes of practice is the idea that \( KA_g/R \) is such limit. In other words, any structure should not be designed for acceleration below \( S_a = KA_g/R \) because it cannot guarantee the preservation of human life and damage cost will be excessive (Section 2).

### 3.1.2. Cost based optimization criterion

An efficient strategy for defining target reliability values for earthquake design is through a cost based optimization (CBO). This process focuses on finding the optimum value of the vector parameter \( p \) for which the building is financially feasible. The vector parameter \( p \) stands for any measure capable of controlling the risk of failure, such as the dimension of the structural elements, the reinforcement, the quality assurance program during construction, the maintenance program during service and so forth. In this case it refers to the mean earthquake design acceleration. The general objective function for maximization can be expressed as

\[
Z(p) = B - C(p) - D(p),
\]

where \( B \) is the benefit derived from the structure, which was assumed to be independent of the vector parameter \( p \); \( C(p) \) is the cost of design and construction; and \( D(p) \) the expected failure cost. If systematic reconstruction of the structure after failure is considered; and assuming a constant benefit (i.e. \( b = \beta C_0 \)), then, for the case of earthquakes in which the events follow a Poissonian process with occurrence rate \( \lambda \) and where failures can occur independently with probability \( P_f(p) \), Hasofer and Rackwitz [9] have shown that the objective function for optimization can be expressed as (see [16]; for a full development of the theory)

\[
Z(p) = \frac{b}{\gamma} - C(p) - (C(p) + H) \frac{P_f(p)\lambda}{\gamma},
\]

where \( H \) is the cost of failure; \( \gamma \) is the annual discount rate corrected for inflation or deflation and averaged over sufficiently long periods; and \( \lambda \) the rate of failure. The model assumes that if the
structure fails, it will be reconstructed and taken up to the reliability standards it had before the event. Therefore, the acceptability criterion for systematic reconstruction is

\[
\frac{b}{\gamma} - C(p) - (C(p) + H) \frac{p_f(p)}{\gamma} \geq 0,
\]

where \( H \) is the cost of losses, which could be further divided into, for example, material and human losses (i.e., \( H = H_M + H_F \)). The lower bound is then defined by the value of the objective function at the optimum, \( Z(p^*) \). Thus, the corresponding probability will be \( p_f(p^*) \). On the other hand, the upper limit can be defined by the value of the objective function for which \( Z(p) = 0 \). In this case, the probability of failure can be expressed as [19]

\[
p_f(p^*) \leq \frac{\left( \frac{C_0}{C(p)} - \gamma \right)}{(1 + H(p^*)/C(p^*))^\lambda}.
\]

The values of vector parameter \( p \) corresponding to the upper and lower limits have to be defined for each combination of \( C(p) \) and \( H(p) \), respectively. An example of the ALARP region for \( \lambda = 4.5, \beta = 0.05 \) and \( \gamma = 0.02 \) is shown in Fig. 3. The bounds depend on benefit and interest rates and on the specific stochastic problem at hand. The way in which the ALARP region is computed shows that the region can change for different contexts [15].

3.1.3. Comparison of strategies for defining the ALARP region

Both strategies described in foregoing sections are compared in Fig. 4. This figure shows the probabilistic side of the spectrum (for a specific sector which depends on the structural vibration period, \( T \)) relating the probability that the spectral acceleration is exceeded and the design acceleration. The upper limit of both ALARP regions are defined by the spectral acceleration with

![Fig. 3. Example of the ALARP region.](image-url)
$R = 1$, while the lower limit corresponds to the design acceleration $R > 1$. However, in terms of probability, the CBO approach limits are defined by value of probability for which $0 < Z(p) < Z(p_{opt})$. This approach depends highly on the project characteristics. On the other hand, in the performance design approach the range of probabilities is defined by the limit probability values defined in Fig. 2 (e.g., $1.38 \times 10^{-2}$, $4.04 \times 10^{-4}$). Note that in order to define this range, many other performance levels can be defined. Some studies have shown that probability values that are defined based on the CBO approach are usually higher than those defined by the structural performance approach [10]. Furthermore, for highly developed countries design acceleration values are higher than for low socio-economic contexts [20].

3.2. Minimum structural performance level (lower bound)

What has been called design acceleration in Fig. 4 and Section 3.1 is a limit to excessive damage and life safety and has been taken as the lower limit of the ALARP Region. In structural terms, FEMA [23] describes the life safety structural performance level of, for instance, concrete frames as “extensive cracking and hinge formation in ductile elements limited cracking and/or splice failure in some non-ductile columns, severe damage in short columns... 2% transient drift and 1% transient drift.”

![Fig. 4. Comparison of strategies for defining the ALARP region.](image-url)
permanent drift”. For the case of partitions, this level is described as “Distributed damage; some severe cracking, crushing and racking in some areas”. On the whole, the expected post-earthquake damage state to ensure life safety requires the structure to remain stable with a significant reserve capacity and hazardous non-structural damage controlled. This has to be provided by a proper combination of the design probability and a reduction factor \( \xi/R \), where \( \xi \) is a constant that will be later discussed.

In most codes of practice, overall structural safety requirements combine factor \( R \) with a control of deformation, in particular on the structural inter-story drift via the deflection amplifying factor \( C_d \). This factor allows certain structural and non-structural damage by using the stored energy dissipation capacity of the structure. Experience has shown that even if the design acceleration is exceeded, it cannot be asserted that the collapse of the building will follow. In fact, design requirements such as redundancy and safety factors used for the design of individual elements provide additional resistant capacity to the building to avoid collapse.

If the main objective of the design is taken as the preservation of life, it is necessary to determine the value of acceleration for which there is at least one person killed if it is exceeded. This can be performed via strength or deformation; the former approach is usually called force-controlled while the latter deformation-controlled. It is then proposed that the design spectrum Eq. (1) can be redefined as

\[
S_a(T, S, I, p_t) = \frac{K}{R} \xi \cdot A_g(p_t),
\]

where \( \xi \) is a factor that affects the design acceleration \( S_a \), taking it to the life safety structural performance level. Statistics of past events show that typical values of \( \xi \) are between 0.75 and 0.9. Although economic losses are also important and should not be left aside, this approach allows a direct comparison with common causes of death. Besides, it can also be used as part of the LQI method which takes into account the Social Cost of Saving Lives (Section 3.2.2) and defines different design criteria depending on the socioeconomic characteristics of the population. There are alternative ways for defining factor \( \xi \): (1) computing probability of being killed in case of an earthquake; and (2) considering the cost of saving lives.

### 3.2.1. Probability of being killed in an earthquake

Computing the probability of actually being killed in a building in case of an earthquake is a difficult task which depends on many factors which are difficult to model. Death tolls are highly variable from one earthquake to another, and data documenting occurrences of life loss in earthquake are poor [3]. Coburn and Spence [4] argues that for a given class of building \( b \), the number of people killed in an earthquake event, given that the building structure has failed, can be expressed as

\[
M_b = C[M_1M_2M_3(M_4 + M_5(1 - M_4))],
\]

where \( C \) is the total number of collapsed structures; \( M_1 \) is the population per building; \( M_2 \) is the occupancy at the time of the earthquake; \( M_3 \) is the occupants trapped by collapse; \( M_4 \) the injury distribution at collapse; and \( M_5 \) mortality post-collapse. Out of all \( M \) factors described in Eq. (11), \( M_3 \) and \( M_4 \) depend on the earthquake intensity and vulnerability curves. All others are functions of the social factors and the logistics of rescue operations. Following typical values investi-
gated by Coburn and Spence [4] for urban areas in Europe, \(M_1 \approx 2.5, M_2 \approx 0.75, M_3 \approx 0.6, M_4 \approx 0.35, M_5 \approx 0.8\), it is possible to compute \(M_b = 0.976 C\). Therefore, the probability of being killed given that the structure has collapsed or is severely damaged can be computed as

\[
p_k = p(F | A_g \geq S_a) = \frac{M_b}{M_s, C} = \frac{0.976 C}{2.5 C} = 0.39, \tag{12}
\]

where \(p(F | A_g \geq S_a)\) is the likelihood of dying given that the ground motion acceleration has exceeded the design acceleration. Therefore, the probability of actually being killed would be

\[
p_{killed} = p(F | A_g \geq S_a)p(A_g \geq S_a). \tag{13}
\]

Note that the last term in Eq. (13) corresponds to the new dimension of the design spectrum. Then, for the example described in Eq. (12), and \(p(A_g \geq S_a) = 1 \times 10^{-4}\), \(p_{killed} = 3.9 \times 10^{-5}\). Statistical data have shown that gross fatality rate is in the order of \(10^{-4} \text{–} 10^{-5}\). For 21 out of the 44 strongest earthquakes in the USA in the last century, the average fatality rate is \(5.5 \times 10^{-5}\); Similarly, Durkin [6] reports fatality rates for Northridge, \(3.2 \times 10^{-5}\); San Fernando, \(4.5 \times 10^{-5}\); and Loma Prieta, \(1.3 \times 10^{-5}\).

It is then suggested that the design acceleration (i.e., \((K/R)\). \(A_g(p_f)\)) can be reduced by a factor \(\xi\) taking into account life losses and severe damage; it is defined as

\[
\xi = \delta \cdot p_k, \tag{14}
\]

where \(\delta\) is a constant that accounts for all those factors that provide additional resistance to the structure and are not explicitly formulated (e.g., redundancy). Note that eventually \(\delta\) may also reflect some special characteristics that are not favorable to the structural resistance. \(\delta\) varies between \(0 < \delta < 1/p_k\). If \(\delta \rightarrow 1/p_k\), \(\xi_{\text{max}} = 1\), and the design acceleration is computed as \(S_a = (K/R)A(p_f)\); if \(\delta \rightarrow 0\), \(\xi_{\text{min}} = 0\) and \(S_a = 0\), which means that the structural characteristics are inappropriate to withstand earthquakes and any ground motion will cause its collapse and the death of all the inhabitants. On the whole, if \(\delta > 1\) the structural system has been provided with additional features to withstand earthquakes and save lives (e.g., redundancy); on the contrary, if \(\delta < 1\) structural deficiencies are important and resistance is inadequate.

The \(\delta\) factor can be assimilated to the reliability/redundancy factor, \(\rho\), defined by the Uniform Building Code (UBC, 1997) based on the extent of structural redundancy inherent in the lateral force-resisting system. The earthquake redundancy factor was first proposed in [2] based on the so-called lines of vertical strength, and deformation of the frame in each principal direction of a building. It is then suggested that a building with less than four lines of strength and deformation, should be penalized (e.g., \(\delta < 1\)) by requiring that higher forces be used for design (e.g., \(\delta = 0.71\) for two lines and \(\delta = 0.86\) for three lines). This factor is defined as

\[
\rho = \max(\rho_x) = \max \left(2 - \frac{6.1}{r_{\text{max}} \sqrt{A_x}}\right), \tag{15}
\]

where \(\rho_x\) corresponds to the \(\rho\) calculated at story “\(x\)” of the structure, \(r_{\text{max}}\) is the ratio of the design story shear resisted by the single element carrying the most shear force in the story to the total story shear for a given direction of loading; and \(A_x\) is the floor area (m²) of the diaphragm level immediately above the story. In general, for all structural types defined by the NEHRP [15], the \(\rho\) factor varies between 1 and 1.25.
There has been much criticism on the rationale behind this provision; in particular, given the large uncertainty of seismic loading and structural resistance, the redundancy of a structural system cannot be treated satisfactorily without careful consideration of the uncertainty. Therefore, a better approach for defining redundancy has been proposed by Wen [22]. The so-called uniform-risk redundancy factor $R_R$ is based on the system’s reliability as the ratio of the system capacity against the incipient collapse. In terms of the spectral acceleration, it corresponds to the ratio between the 50-year incipient collapse probability $p_{50}$ and the allowable probability $p$. Based on this definition, $R_R$ is smaller to 1 for a system with inadequate reliability/redundancy. Then, the design seismic force is reduced by a factor $R$ multiplied by $R_R$. Therefore, factor $\delta$ can be redefined as $\delta = 1/R_R$.

3.2.2. Life saving cost strategy

The factor $\xi$ can be also obtained from the Life Quality Index (LQI) [12,14], which is a recently developed concept related to what economist call value of statistical life, which is higher than the so called human capital, i.e., the lost earnings in case of premature death. The value of statistical life has nothing to do with the amounts which have to be paid to the surviving dependents after an event as compensation by insurance or the social system. It is the monetary value a society should invest to save an anonymous life from a life-threatening hazard [17]. These ideas lead to the societal cost of saving life (SLSC), or the so-called implied cost of averting a fatality (ICAF), which is defined as [17]

$$SLSC = \int_0^{a_g} LSC(e(a))h(a,n) \, da,$$

where $LSC(e(a))$ is what has to be invested into technical projects for safety related actions at the decision point ($t=0$); and $h(a,n)$ is the density of the age distribution of the population with a population growth rate $n$. Several models can be defined for the mortality regime, $h(a,n)$: (1) stationary population ($n=0$); (2) small changes in crude mortality that distributes equally as a constant at all ages; and (3) a case within which age-dependant mortality changes proportional to the age distribution. More details on the derivation of this factor and the applications can be found in [17]. According to this theory, the life saving cost of a technical project with $N_{PE}$ potential endangered persons is

$$H_F = SC\cdot k \cdot N_{PE},$$

where $k$ is a proportionality constant that relates changes in mortality, $dm$, to changes in the failure rate, $dh$, i.e., $dm = k \cdot dh(p)$; where $p$ is the vector design parameter [17], which in this case, is the design acceleration. The factor $k$ can be interpreted as the likelihood of being killed in case of an earthquake. Then, if $N_{PE}$ is obtained from Eq. (11), i.e., $C \cdot M_1 = N_{PE}$, and the total number of fatalities is $N_F = C \cdot [M_1 M_2 M_3 (M_4 + M_5 (1 - M_4))]$:

$$k = \frac{N_F}{N_{PE}} = \left[\frac{M_1 M_2 M_3^2 (M_4 + M_5 (1 - M_4))}{M_1}\right] = p_k.$$

(18)

For the case of earthquakes, Rackwitz [17] has proposed a rough approximation for estimating $p_k$ as a function of the acceleration $a$ in the form $p_k(a) = 1 - \exp(-\eta a)$, where $\eta$ is a constant $\eta < 1$. For instance, for $a = 2 \, m/s^2$ and $\eta = 0.1$, $p_k = 0.18$; similarly, for $a = 10 \, m/s^2$, $p_k = 0.63$. In the for-
mer case, in order to obtain a value of $\xi = 1$ it is necessary that $\delta = 5.55$ ($R_R = 0.182$); while, in the latter case a value of $\delta = 1.58$ ($R_R = 0.633$) is required. Based on Eq. (17) (i.e., life saving cost of a technical project), Eq. (14) can be transformed into

$$\xi = \delta \frac{H_F}{\text{SLSC} \cdot N_{PE}}.$$  \hspace{1cm} (19)

The factor $\xi$ can be defined in terms of $H_F$ since all other values are constants defined for a given social and structural context. In this case, losses caused by injuries are neglected. It can be assumed, for example, $\delta = 1.25$, SLSC$_\text{USA} = 8.7 \times 10^5$ (USA); $N_{PE} = 10$ and $H_F = 5.5 \times 10^6$; then $\xi = 0.79$. It is important to mention that only small variations of $H_F$ with respect to SLSC are expected; otherwise, the equality principle would be severely violated. Since the discussion in this paper is about code regulations and social acceptability of risk, a given society or sector of a society may decide to increase or reduce the value of $H_F$ depending on the hazard characteristics (e.g., frequency, death tolls) or on specific consideration of the people involved. In addition, under comparable conditions, a lower amount of risk reduction would be required, for instance, in a developing country for increasing investment.

Fig. 5 presents the variation of factor $\xi$ as a function of $H_F$ for different scenarios. It is observed that as the number of fatalities increases, higher values of $H_F$ are required to keep $\xi$ close to 1. Further analysis of these results can be used as support for defining appropriate target reliabilities for different sectors of society or for special activities which may have an important effect on society. Eventually, under appropriate regulation and law enforcement, this approach may be used to define safety as a quality of a building; in other words, it may become a commodity which can be traded. Nevertheless, it should not be used for defining insurance policies or for any compensation to the families of victims.

4. Comparative analysis of acceptability criteria

Assume that a four story reinforced concrete structure is designed for $R = 5$ and $I = 1.1$. The local emergency response conditions suggest that, in case of collapse, the likelihood of being killed is $p_k = 0.39$. If the probabilistic model of the ground motion is as described in Section 2.2, the problem is to define the design acceleration by using different criteria.

The first structural vibration mode can be approximated to $T \approx 0.4$; therefore, the design acceleration has to be defined for the intermediate sector of the design spectrum (e.g., $0.48s < T < 2.4s$ for the Colombian code of practice) and $K/R = (2.5)(1.1)/(5) = 0.55$. If the design considers the actual probability of being killed in case of an earthquake, the probabilistic spectrum has to be modified by the factor $\xi$ so that the comparison with risk acceptability is appropriate. Taking that $\delta = 1.92$ ($R_R = 0.521$), then $\xi = (1.92)(0.39) = 0.75$. Note that for this case, $\xi < 1$, and the design acceleration has to be reduced further from $(K/R)A(p_f)$. The design acceleration values for all performance states defined in Section 2.1 are shown in Table 1.

Alternatively, Wen et al. [24] have defined seismic performance goals for different structural types as shown in Table 2. This table also shows the design spectral acceleration values for the conditions described in this section.
The purpose behind the idea of defining life safety requirements is to have a reference framework that can be compared with causes of death which are common in any society. Fig. 6 presents different criteria taken from annual risk of death by different causes in Colombia [21]. By using this approach, it is possible to design a building structure with the same probability of being killed in a car accident, or of dying by cancer. The design acceleration values based on comparison with common causes of death in Colombia are also shown in Table 3.

The factor \( n \) may also be defined in terms of the social cost of saving life. Based on Eq. (19), the design acceleration as a function of various life saving cost sums for all fatalities in a building and for the average probability of exceedence of the design earthquake prescribed by the code of practice is shown in Table 4. It is observed that the factor \( \xi \) depends on the number of fatalities but mostly on the life saving cost value considered.

Finally, an optimization can be performed to define the optimum values of the mean design acceleration to guarantee the maximum economic benefit Eq. (7). Based in the work by Sán-

![Fig. 5. Factor \( \xi \) in terms of the life saving cost in case of structural collapse for USA.](image)

<table>
<thead>
<tr>
<th>Performance state</th>
<th>Return period</th>
<th>( p(S_a &lt; A_g) )</th>
<th>( S_a ) (m/s(^2)) ( \xi = 0.75 )</th>
<th>( S_a ) (m/s(^2)) ( \xi = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fully operational</td>
<td>72</td>
<td>( 1.38 \times 10^{-2} )</td>
<td>0.27</td>
<td>0.338</td>
</tr>
<tr>
<td>Basic operation</td>
<td>225</td>
<td>( 4.44 \times 10^{-3} )</td>
<td>0.45</td>
<td>0.599</td>
</tr>
<tr>
<td>Life safety</td>
<td>475</td>
<td>( 2.10 \times 10^{-3} )</td>
<td>0.62</td>
<td>0.829</td>
</tr>
<tr>
<td>Collapse prevention</td>
<td>2475</td>
<td>( 4.04 \times 10^{-4} )</td>
<td>1.045</td>
<td>1.391</td>
</tr>
</tbody>
</table>
Table 2
Seismic performance goals for different types of structures

<table>
<thead>
<tr>
<th>Performance state</th>
<th>( P_{\text{Goal}} )</th>
<th>( S_a ) (m/s²) ( \zeta = 0.75 )</th>
<th>( S_a ) (m/s²) ( \zeta = 1.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>General use</td>
<td>0.001</td>
<td>0.82</td>
<td>1.002</td>
</tr>
<tr>
<td>Important facilities</td>
<td>0.0005</td>
<td>1.013</td>
<td>1.346</td>
</tr>
<tr>
<td>Essential facilities</td>
<td>0.0002</td>
<td>1.267</td>
<td>1.686</td>
</tr>
<tr>
<td>Moderate hazard facilities</td>
<td>0.0001</td>
<td>1.468</td>
<td>1.954</td>
</tr>
<tr>
<td>High hazard facilities</td>
<td>0.00001</td>
<td>2.208</td>
<td>2.937</td>
</tr>
</tbody>
</table>

Fig. 6. Design acceleration values \((K/R = 0.55, \zeta = 0.75)\) and comparison with common causes of death in Colombia. BRD – respiratory related diseases; TA – traffic accidents; DSRD – digestive related diseases; ID- infectious diseases; COP-value demanded by the Colombian code of practice.

Table 3
Different design acceleration values as function of the cause of death

<table>
<thead>
<tr>
<th>Risk taken as reference</th>
<th>( S_a ) (m/s²) ( \zeta = 0.75 )</th>
<th>( S_a ) (m/s²) ( \zeta = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Herat attack</td>
<td>0.229</td>
<td>0.296</td>
</tr>
<tr>
<td>Homicide</td>
<td>0.278</td>
<td>0.370</td>
</tr>
<tr>
<td>Cancer</td>
<td>0.385</td>
<td>0.517</td>
</tr>
<tr>
<td>Respiratory related diseases</td>
<td>0.429</td>
<td>0.572</td>
</tr>
<tr>
<td>Traffic accident</td>
<td>0.516</td>
<td>0.691</td>
</tr>
<tr>
<td>Digestive system problems</td>
<td>0.612</td>
<td>0.818</td>
</tr>
<tr>
<td>Infectious diseases</td>
<td>0.720</td>
<td>0.960</td>
</tr>
<tr>
<td>NSR-98 – Col. code of practice</td>
<td>0.623</td>
<td>0.818</td>
</tr>
</tbody>
</table>
chez-Silva and Rackwitz [20], optimum values can be obtained for three different social climates based on LQI concepts. Results for high (USA, Canada, Japan, Western European Countries), moderate (Latin-American), and low (African, some Caribbean countries) socioeconomic climates are showed in Table 5. For the local seismic conditions, the design acceleration specified in the code of practice is $S_a = K A_g = (0.7) A_g = 1.1 \text{ m/s}^2$. Results in Table 5 also show that design acceleration values should be differential depending upon the socioeconomic context.

### Table 4
Definition of design acceleration values based on the LQI for $\delta = 1.92$, $R = 5$, $p(S_a < A_g) = 2.01 \times 10^{-3}$ ($p = 1/T = 1/475$)

<table>
<thead>
<tr>
<th>$H_F^a$</th>
<th>$N_F$</th>
<th>$\xi$</th>
<th>$S_a$ (m/s$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.40 SLSC</td>
<td>3</td>
<td>0.256</td>
<td>0.21</td>
</tr>
<tr>
<td>0.60 SLSC</td>
<td>3</td>
<td>0.384</td>
<td>0.31</td>
</tr>
<tr>
<td>0.80 SLSC</td>
<td>3</td>
<td>0.512</td>
<td>0.43</td>
</tr>
<tr>
<td>1.00 SLSC</td>
<td>3</td>
<td>0.640</td>
<td>0.54</td>
</tr>
<tr>
<td>2 SLSC</td>
<td>10</td>
<td>0.384</td>
<td>0.31</td>
</tr>
<tr>
<td>5 SLSC</td>
<td>10</td>
<td>0.960</td>
<td>0.80</td>
</tr>
<tr>
<td>7 SLSC</td>
<td>10</td>
<td>1.344</td>
<td>1.10</td>
</tr>
<tr>
<td>10 SLSC</td>
<td>10</td>
<td>1.920</td>
<td>1.52</td>
</tr>
</tbody>
</table>

$^a$ SLSC$_{USA} = 8.5 \times 10^5$.

### Table 5
Comparison of optimum target design criteria among different socioeconomic climates

<table>
<thead>
<tr>
<th>Socioeconomic context</th>
<th>Optimum design acceleration (m/s$^2$)</th>
<th>$p_F$</th>
<th>$\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>1.55</td>
<td>$7.624 \times 10^{-5}$</td>
<td>1.4</td>
</tr>
<tr>
<td>Moderate</td>
<td>1.06</td>
<td>$4.18 \times 10^{-4}$</td>
<td>0.96</td>
</tr>
<tr>
<td>Low</td>
<td>0.78</td>
<td>$1.15 \times 10^{-3}$</td>
<td>0.71</td>
</tr>
</tbody>
</table>

5. Effects of competitiveness in third world countries: Colombian case

5.1. Basic housing problem in Colombia

In third world countries, housing for the poorest sector of society is a priority. It is important not only because it is a basic need, but also because it is one of the main sources of employment in economies where technology and heavy industry are not dominant. This means that construction is highly sensitive to variations in the economy and small cost benefits have important effects on the sector. In general, housing for poor sectors and transport infrastructure are essential as they are some of the main attractors for investment.

Social statistics show that the low income housing deficit in Colombia in the year 2000 was 1,880,529 units which correspond to 26.5% families. Recent studies and housing developing programs [7] for the city of Bogotá have shown that the average cost of building a house for the poorest sectors of the population has a cost somewhat around US$6000/house. This means that covering the whole deficit costs about US$11.28 billion and some additional 5 billion for the basic
infrastructure. Then, based on the discussion in previous sections, the question of how much money can be saved, if different safety criteria are applied, becomes relevant.

### 5.2. Acceptability criteria and cost analysis

Echeverry et al. [7] studied different configurations for low income housing buildings and investigated costs in detail in every case. In order to estimate the differences in cost by using different acceptability criteria, a multi-familiar three story reinforced concrete frame building was considered as a general case. It has a square plan shape (12 m × 12 m) with inter-story height of 3 m. The structure was designed according to safety standards defined in the Colombian code of practice, which are essentially the same as those specified by the NEHRP.

In Colombia, the structural cost of buildings in the wealthiest sectors of society is between 15% and 25% of the total cost of the building while for the poorest sectors this value is between 40% and 50%. In bridges and other types of transport infrastructure these values may reach 70% and 80%. Based on statistical data, it was assumed that the cost of the structural system is 30% of the total cost of the building. Results show that, as expected, the cost of the structural system grows as the design acceleration increases. Table 6 presents the cost of building all of the housing required by using different acceptability criteria.

Columns 2 and 3 are the cost of construction of the structural system of all housing needed for different reference acceptability criteria, while columns 4 and 5 show the percentage change of the cost with respect to the cost if the structure is designed for the specifications of the code of practice. For example, if the structure is designed for the same probability of dying of heart attack, it is possible to reduce the cost in 12.5% for \( \xi = 1 \) and 10.5% for \( \xi = 0.75 \). For the Colombian case, a 5% reduction in the total cost implies saving some US$814 million, which is about 0.1% of the GDP, a considerably important amount.

<table>
<thead>
<tr>
<th>Risk taken as reference</th>
<th>US$(bill) ( \xi = 0.75 )</th>
<th>US$(bill) ( \xi = 1 )</th>
<th>( %^a \xi = 0.75 )</th>
<th>( %^a \xi = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>General use [24]</td>
<td>2.337</td>
<td>2.429</td>
<td>104.6</td>
<td>104.0</td>
</tr>
<tr>
<td><strong>Performance states:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fully operational</td>
<td>2.028</td>
<td>2.072</td>
<td>90.8</td>
<td>88.7</td>
</tr>
<tr>
<td>Basic operation</td>
<td>2.139</td>
<td>2.222</td>
<td>95.7</td>
<td>95.1</td>
</tr>
<tr>
<td>Life safety</td>
<td>2.233</td>
<td>2.342</td>
<td>99.9</td>
<td>100.2</td>
</tr>
<tr>
<td>Collapse prevention</td>
<td>2.450</td>
<td>2.621</td>
<td>109.6</td>
<td>112.2</td>
</tr>
<tr>
<td><strong>Common causes of death:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heart attack</td>
<td>2.000</td>
<td>2.045</td>
<td>89.5</td>
<td>87.5</td>
</tr>
<tr>
<td>Homicide</td>
<td>2.033</td>
<td>2.092</td>
<td>91.0</td>
<td>89.5</td>
</tr>
<tr>
<td>Cancer</td>
<td>2.101</td>
<td>2.177</td>
<td>94.0</td>
<td>93.2</td>
</tr>
<tr>
<td>Respiratory related diseases</td>
<td>2.127</td>
<td>2.207</td>
<td>95.2</td>
<td>94.5</td>
</tr>
<tr>
<td>Traffic accident</td>
<td>2.176</td>
<td>2.271</td>
<td>97.4</td>
<td>97.2</td>
</tr>
<tr>
<td>Digestive system problems</td>
<td>2.229</td>
<td>2.336</td>
<td>99.7</td>
<td>100.0</td>
</tr>
<tr>
<td>Infectious diseases</td>
<td>2.286</td>
<td>2.408</td>
<td>102.3</td>
<td>103.1</td>
</tr>
<tr>
<td>Code of practice</td>
<td>2.235</td>
<td>2.336</td>
<td>100.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>


\( ^a \) Comparative percentage with respect to the code of practice.
6. Conclusions

Reviewing the criteria for defining acceptability is important as long as it provides new insights and solutions on relevant aspects of society. New trends in earthquake engineering have shown that design should include the uncertainty as part of the process of defining the design acceleration. This paper proposes a probabilistic design spectrum which includes the probability that the design acceleration is exceeded. The probabilistic spectrum provides an alternative for dealing with cases which are very difficult to manage nowadays. It is a flexible designing tool that may be used to propose new risk transfer methodologies. Besides, this may be also important for infrastructure development, since it can make designs more flexible and, if well coordinated with rehabilitation and maintenance programs, it can lead to more efficient solutions with higher returns on investment. Although numerically it can be easily obtained, the use of such design criterion requires defining boundaries on the selection of failure probabilities and an adequate guide for the user. The discussion about the selection of appropriate probability values (i.e., acceptability criteria) has many sides, and there is clearly a difficulty in defining a unique criterion, which fits all contexts. An example of this alternative has been illustrated with a large scale housing problem. Results have shown the viability of the probabilistic design spectrum and the consequences of this flexibility in the development of low income housing in third world countries.

References